The Evolution of Markets and the Revolution of Industry: A Quantitative Model of England's Development, 1300-2000*

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Abstract

This paper argues that an economy's transition from Malthusian stagnation to modern growth requires markets to reach a critical size, and competition to reach a critical level of intensity. By allowing an economy to produce a greater variety of goods, a larger market makes goods more substitutable, raising the price elasticity of demand, and lowering mark-ups. Firms must then become larger to break even, which facilitates amortizing the fixed costs of innovation. We demonstrate our theory in a dynamic general equilibrium model calibrated to England's long-run development and explore how various factors affect the timing of takeoff.

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1 Introduction

Over the last decades a much more complete and accurate picture of the *Industrial Revolution* has emerged on account of detailed data-oriented work by economic historians. Whereas this picture is complex, a number of aspects stand out. First, cost-reducing technological innovation implemented by individuals and firms was essential to industrialization (Landes, 1969, Mokyr, 1990). Second, innovation in consumer products was nearly as common as innovation in production processes, with this *Consumer Revolution* preceding the *Industrial Revolution* (Styles, 2000, Berg, 2002). Third, improving transportation infrastructure, together with a favorable geography, turned Britain into an increasingly integrated marketplace (Szostak, 1991). Fourth, the great inventions of the 18th century were preceded, and later accompanied, by an organizational shift from the cottage industry and putting-out system to the centralized workplace (Szostak, 1989, Berg, 1994).

Although historical work has illuminated the nature of the *Industrial Revolution*, its causes remain, in the words of Clark (2003), "one of history's great mysteries".[1] This paper attempts to demystify the *Industrial Revolution* by putting forth a novel theory that is consistent with the picture provided by economic historians. In this theory, a gradual expansion of the market, coupled with increasing variety of consumer goods and growing firm size, sows the seeds for process innovation, which allows the economy to transition from Malthusian stagnation to modern growth. We show that our theory is empirically plausible by deriving its quantitative implications in a model calibrated to the historical record of England over the 1300-2000 period and by providing micro-evidence for the mechanism that underlies it.

The key mechanism in our theory links market size to innovation through the price elasticity of demand. A larger market allows an economy to sustain a greater variety of goods, making them more substitutable and increasing their price elasticity of demand. As a result, mark-ups drop and competition toughens so that firms must become larger to break even. This facilitates innovation, as bigger firms can spread the fixed costs of R&D over a greater quantity of output. Therefore, only after the market reaches a critical size and competition is strong enough, does innovation endogenously take off and do living standards rise. An *evolution* of markets is thus a precondition for a *revolution* of industry.

We generate this elasticity effect by embedding Lancaster (1979) preferences into a model of product and process innovation. The Lancaster construct, based on Hotelling's (1929) spatial model

[1] This is also the implicit conclusion of Mokyr and Voth (2007) in reviewing the theories of the *Industrial Revolution*.

of horizontal differentiation, assumes that each household has an ideal variety of an industrial good, identified by its location on the unit circle. As goods 'fill up the circle', neighboring varieties become closer substitutes, implying a higher price elasticity of demand and a lower mark-up (Helpman and Krugman, 1985, Hummels and Lugovskyy, 2005). As shown by Desmet and Parente (2010) in a static one-sector model, these preferences imply a positive effect of market size on technological innovation.[2]

Apart from the preference structure, the model is fairly standard and in some aspects simpler than alternative models of the *Industrial Revolution*. In the spirit of Galor and Weil (2000), it includes a farm sector that produces a subsistence agricultural good, and it assumes that parents derive utility from having children. However, in contrast to models that emphasize the role of human capital in economic development, parents do not face a tradeoff between the quantity and the quality of children.[3] Instead, there is a time-rearing cost to children that is lower on the farm than in the city. With these features, the model not only generates a rapid transition from Malthusian stagnation to modern growth, but also a structural transformation with a declining agricultural share, and a demographic transition with population growth initially rising with the advent of industrialization and subsequently falling.

The model works as follows. The subsistence constraint, together with low initial agricultural productivity, implies that the economy starts off with most of its population employed in agriculture. Given that so few people live and work in the city and given the fixed operating cost, only a small number of industrial varieties are produced, implying that goods are not particularly substitutable. Mark-ups are high, and hence, firms are small. As a result, firms do not find it profitable to incur the fixed costs of innovation. However, during this Malthusian phase with stagnant living standards, exogenous increases in agricultural TFP allow for increases in the population and a larger urban base. Eventually, the population reaches a critical size, making industrial firms sufficiently large to warrant process innovation. At this point, firms endogenously lower their marginal costs, and hence, an industrial revolution ensues.

Process innovation in the industrial sector then sets off a demographic transition and a structural transformation. As incomes rise, both rural and urban households have more children,

[2] The constructs of Salop (1979), Gali and Zilibotti (1999), Feenstra (2003), and Ottaviano et al. (2005) all generate this elasticity effect. However, it is not present in the standard Spence-Dixit-Stiglitz construct, although it can be generated by assuming that firms take into account how their decisions affect the aggregate price level (Yang and Heijdra, 1993). Our reasons for using Lancaster is that the elasticity effect is intuitive, the construct is analytically tractable, and it allows for income effects. It is also fairly straightforward to calibrate.

[3] See, e.g., Becker et al. (1990), Galor, and Weil (2000), and Lucas (2002).

because household utility is increasing in the number of children. This leads to accelerating population growth: the first phase of the demographic transition. At the same time, rising incomes relax the subsistence constraint, implying a structural transformation, with a decreasing agricultural employment share. Since the cost of child rearing is higher in the city, urban households have lower fertility than their rural counterparts. With an increasing share of people living in the city, this puts a brake on aggregate fertility. Eventually, this compositional effect dominates the income effect, and the population growth rate slows down: the second phase of the demographic transition.

In the limit, as living standards continue to rise, the subsistence constraint disappears, and the economy converges to constant agricultural and industrial shares of economic activity. Under certain parametric conditions, the population growth rate converges to zero, and the price elasticity of demand approaches a constant. Firm size ceases to increase, and the rate of innovation becomes constant. Thus, the economy converges to a modern growth era with a constant positive growth rate of per capita GDP.

To assess the plausibility of our theory, we calibrate the model to the historical experience of England from 1300 to 2000. More specifically, we restrict the model parameters to match pre-1700 and post-1950 English observations, and then test the model by examining its predictions corresponding to the 1700-1950 period. We find that the model accounts remarkably well for the main features of England's experience during this transition period. In particular, it closely matches England's growth path, its structural transformation, and its demographic transition. The quantitative success of the model, together with independent empirical evidence on the elasticity mechanism provided in Section 2, constitute strong support for our theory.

The literature on the *Industrial Revolution* is extensive. A number of themes in our theory echo back to three older branches of this literature. The first is the *Industrial Organization* school, which views the emergence of large firms with supervised production as the key to the *Industrial Revolution*. Important contributions to this literature are Mantoux (1928), Pollard (1965) and Berg (1994). The second is the *Social Change School*, which equates the *Industrial Revolution* to the development of competitive markets. This view is present in the work of Toynbee (1884), Polanyi (1944) and Thompson (1963). The final branch of this older literature emphasizes demand side factors, in particular, the growth of the home market and the development of consumer demand. Here some of the important papers are Gilboy (1932) and McKendrick (1982).

With respect to the more recent literature, our work is most closely related to unified growth theory, which analyzes the transition from Malthusian to modern growth within a common framework. Some of the important papers in this literature are Kremer (1993), Goodfriend and McDermott (1995), Galor and Weil (2000), Hansen and Prescott (2002), Lucas (2002), and Galor and Moav (2003). These theories emphasize very different mechanisms from ours though. Moreover, they do not model process innovation in the sense of individual firms spending resources to lower their marginal costs, nor do they introduce product innovation in the sense of increasing the variety of consumer goods.[4]

Our paper also relates to the literature that uses model calibration to gain intuition for the causes of the *Industrial Revolution*, particularly, why England was the first country to transit from the Malthusian era. Important papers in this literature are Harley and Crafts (2000), Stokey (2001), Lagerlöf (2003, 2006), and Voightländer and Voth (2006). As these other models are very different from ours, they study the effect of a different set of factors on the timing of England's takeoff. In our experiments, we consider three factors, each of which has been emphasized by other researchers as being important for England's *Industrial Revolution*: agriculture productivity (Schultz, 1968, Diamond, 1997), institutions (North and Thomas, 1973, North and Weingast, 1989), and trade (Findlay and O'Rourke, 2007). Our counterfactuals support the view that each of these factors was important for England's development, perhaps hastening its industrialization by several centuries.

In our theory population size and fixed costs are important determinants of the timing of industrialization. Recently, the empirical relevance of these factors has been questioned by economic historians. Crafts (1995), for example, has criticized population-based theories on account that bigger countries have not grown faster, and both Mokyr (1999) and Mokyr and Voth (2007) have argued against fixed costs on account that most industries in the 18th century were characterized by small firms.

Regarding the population size criticism, we note that in our theory market size, and not population size per se, is the key determinant of an economy's takeoff.[5] While the size of the market depends on a country's total population, it is also affected by transport costs, internal and external trade barriers, and institutions. France clearly had a greater population than England at the start of the 18th century, but evidence in the form of price variations (Shiue and Keller, 2007) and transport costs (Szostak, 1991) strongly suggest that markets in England were much more national

[4] Although some models, such as Goodfriend and McDermott (1995) and Voigtländer and Voth (2006), allow for increasing variety of intermediate goods to capture Smith's (1776) hypothesis that the *Industrial Revolution* was the consequence of greater specialization, final goods producers continue to be perfectly competitive. As a result, these models not only fail to generate an increasing number of consumer commodities, they are also unable to account for the fixed costs of technology adoption, and for the growing size of firms before and during the *Industrial Revolution*

[5] Recent work by Alesina et al. (2004) suggests that size is important for closed economies.

in scope.[6] Regarding the fixed cost criticism, note that in our theory fixed costs need not be large, and therefore, firms need not be particularly large either. Moreover, the fixed costs of innovation are not limited to those incurred by the original inventor; they also refer to the resources used to adapt a workshop or factory to a new production process. Historically, an important fixed cost associated with technology adoption was the cost of overcoming worker resistance (Mokyr, 1990). Also consistent with the existence of fixed adoption costs, Sokoloff (1984) reports that during the first half of the 19th century firms in the U.S. were largest in those areas where mechanization had proceeded fastest.

The rest of the paper is organized as follows. Section 2 provides empirical support for the mechanism put forth in this paper, and hence serves as motivation. Section 3 describes the model and characterizes the optimal decisions of agents. Section 4 defines the equilibrium and shows algebraically that under certain conditions the economy converges to a balanced growth path. Section 5 calibrates the model to the historical experience of England, and considers how agricultural productivity, institutions, and trade affected the date of the economy's take-off. Section 6 concludes the paper.

2 Empirical Motivation

In this section we motivate our theory by presenting empirical evidence for the underlying mechanism by which greater market size leads to more innovation. This mechanism relies on increased variety of consumer goods raising the price elasticity of demand. In response to this change, mark-ups fall, implying larger firms. The increase in firm size, then, strengthens the incentives of firms to lower their marginal costs. Thus, in what follows, we document secular trends in product variety, price elasticity of demand, mark-ups, firm size, and process innovation.

Product innovation Consistent with the historical record, our model leads to both product and process innovation. A growing body of literature argues that product innovation was every bit as important to the *Industrial Revolution* as process innovation. The creation of new consumer goods, and the increase in varieties, was an essential feature of the *Industrial Revolution* and the period leading up to it. Berg (2002), for example, in analyzing the nature of British patents for

[6] China is another relevant comparison. Kelly (1997) suggests that China's growth during the Sung Dynasty (9th through 12th centuries) was the result of a creation of a national waterway network that expanded markets. For the Qing Dynasty (17th and 18th centuries) the population of China's most vibrant region, the Lower Yangtze, did not change much despite overall population growth in the country, and the region's institutions became worse for business according to Ma (2006).

the period 1627-1825 in a subset of industries, including metal wares, glass, ceramics, furniture and watches, found that over one-quarter of the 1,610 patents specified new products or variations of existing ones. In a narrower study, Griffiths et al. (1992) document that roughly half of the 166 patented and non-patented improvements in the textile industry between 1715 and 1800 concerned product innovation. Similarly, De Vries (1993), using records from probate inventories, documents increasing variety in household durables through the 18th century, despite relatively stagnant wages.

The increase in new consumer goods and varieties is not a post 17th century phenomenon. Weatherwill (1988) argues that the *Consumer Revolution* peaked between 1680 and 1720. Referring to the 1500-1700 period, Styles (2000) lists a number of products that were either entirely English inventions or dramatic remodeled goods from other societies, such as pocket microscopes, drinking cups made from lead glass, and watches. Other new products for English consumers, such as Delftware plates, Venetian glass, and upholstered chairs, originated in Europe. Still others, such as porcelain, tea, tobacco, sugar, lacquered cabinets, and painted calicos, came from Asia and the New World.

Price elasticity of demand. Our mechanism is based on larger markets generating higher price elasticities of demand. The most direct support for our theory would be evidence of a secular rise in the price elasticity of demand. Unfortunately, time series estimates for the price elasticity of individual products do not exist. Even in the cross-section, such estimates are uncommon. We know of only two such studies, both consistent with our model. These are Barron et al. (2008) who compute price elasticities in U.S. gasoline markets and find that larger markets are associated with more elastic demand, and Hummels and Lugovskyy (2005) who document that import demand in larger markets is more responsive to changes in trade costs.

Mark-ups. Our mechanism implies a secular decline in mark-ups associated with the increase in the price elasticity of demand as markets expand. Estimates of mark-ups are more readily available, although most studies are contemporary. Short-run studies in the context of business cycles and trade liberalizations strongly suggest that the mark-up is inversely related to market size. Within the trade literature, Tybout (2003) documents that mark-ups generally fall following liberalization, and within the business cycle literature, Chevalier and Scharfstein (1996) and Chevalier et al. (2003) find that mark-ups are countercyclical both at the aggregate and the industry level. To our knowledge, there is only one long-run study on mark-ups. Ellis (2006) estimates mark-ups in the

United Kingdom for the period 1870-2003 and finds a 67 percent decline in this 134 year period. Taken together, these studies support the negative relation between market size and mark-ups implied by our mechanism.

Firm size. Another implication of our mechanism is that firm size increases with market size. Here, studies are much more abundant, and supportive. There is a large and extensive literature that documents increases in establishment size since industrialization for both England and the United States. For example, Lloyd-Jones and Le Roux (1980) document that the median number of workers in cotton firms in Manchester more than tripled between 1815 and 1841. In the case of pig iron, Feinstein and Pollard (1988) report that in England production per furnace increased from 400 tons in 1750 to 550 in 1790. Using data from the U.S. Census of Manufacturing, Sokoloff (1984) and Atack et al. (1999) find more of the same in manufacturing industries over the 19th century, while Granovetter (1984) documents this pattern continued into the 20th Century.

Sokoloff's (1984) study is particularly relevant because it shows that an increase in firm size was occurring prior to 1860, considered the starting year of the US *Industrial Revolution*. In addition, he finds that firms also grew in size in industries that did not mechanize, such as tanning, hats, boots and shoes. This is consistent with our theory, which predicts that the increase in firm size predates the economy's take-off. The Sokoloff study further supports our mechanism by uncovering a positive correlation between market size, firm size, and the level of industrialization. More densely populated areas had larger firms within a given industry. For example, in the wool textiles industry in 1850 average firm size was 38.7 in New England, compared to 14.5 in the Mid Atlantic, and 6.5 in the rest of the country. Additionally, more densely populated areas industrialized first. Finally, whereas artisan shops coexisted with factories during the first half of the 19th century, artisan shops were located in rural areas with low population density and high transportation costs.

There do not seem to be hard numbers of firm size in England predating the *Industrial Revolution*. Nevertheless, a burgeoning literature on *proto-industrialization* suggests that firm size increased substantially before the onset of England's takeoff. Proto-industrialization refers to the period between 1500 and 1700, when non-agricultural goods were produced in the countryside for large regional, and even international, markets. It is associated with the rise of the *putting-out* system, consisting of merchant capitalists who would sell inputs to rural households and buy finished good in return. Under this system merchants controlled and centralized a number of activities, such as marketing and finance, whereas production was decentralized to rural households. If one interprets the putting-out network as an organization, then the size of organizations was clearly increasing before the *Industrial Revolution*. The only difference, compared to the factory system, is that not all tasks were performed under the same roof.

Some centralization of production did occur in this period, however, although it tended to be limited to specific parts of the production process. For example, in the cotton industry spinning became mechanized and centralized, when weaving was still being done by cottage industries. Similarly, the printing of fabrics became centralized early on. Calico printing workshops were proto-factories, and some of the leading calico printers were associated with the introduction of mechanized spinning and weaving. In the woolen industry, the artisan system was retained by clothiers using cooperative mills that centralized part of the production process. In the knitting industry, apart from sophisticated putting out networks, the industry boasted centralized workshops from early in the eighteenth century. Large workshops employing over forty parish apprentices existed in Nottingham as early as the 1720s. By the time that Hargreaves and Arkwright went to Nottingham, the concentration of labor in factories was a fairly familiar idea (Berg, 1994).

Many researchers, particularly Mendels (1972), argue that the rise of the cottage industry was a critical step in the eventual industrialization of the British economy. Indeed, Mendels claims that the proto-industrialization period was a critical transition phase from the feudal world of the Middle Ages to the capitalist world of the modern era. Our work compliments this area of research. In our theory increases in firm size not only predate the start of the *Industrial Revolution*, they are necessary for it to occur.

Firm size and process innovation. Lastly, the final link in our mechanism is the one from firm size to process innovation. The idea that firm size facilitates process innovation has a long history in economics, going back as far as Schumpeter (1942). There is much empirical evidence supporting this view. For example, Atack et al. (2008) find that larger firms were more likely to use steam power in the 19th century. Hannan and McDowell (1984) reach a similar conclusion when analyzing the relationship between the size of banks and the adoption of ATMs in the 1970s. In terms of R&D expenditures, Cohen and Klepper (1996) find that they rise with firm size, with a greater share being allocated to process innovation.[7]

[7] Taken together, Sokoloff (1984) showing that firms were larger in larger markets, and Sokoloff (1988) showing that patenting activity was greater in larger markets, support the positive link between firms size and innovation.

3 The Model

In this section we describe the structure of the model economy. The economy consists of one country, with a rural and an urban area, and zero transportation costs.[8] Time is discrete and infinite. There are three sectors: a farm sector, an industrial sector, and a household sector. The farm sector is perfectly competitive and produces a single non-storable consumption good. The farm technology uses labor and land and is subject to exogenous technological change. The industrial sector is monopolistically competitive and produces a finite set of differentiated goods, each of which has a unique address on the unit circle. There is both product and process innovation in the industrial sector. The household sector consists of one-period lived agents, each of whom derives utility from consumption of the agricultural good, consumption of the differentiated industrial goods, and children. For each household, there is a variety of the differentiated good that it prefers above all others. Households earn income by either working in the farm sector or the industrial sector. In addition to working, households use their time to rear children, who constitute the household sector in the next period. In what follows we describe the model structure and the relevant optimization problems encountered by agents in each sector.

3.1 Household Sector

Endowments. At the beginning of period t there is a measure N_t of households. Each household is endowed with one unit of time, which it uses to rear children and to work in either the farm or the industrial sector. There are no barriers to migration, so that a household is free to work in either sector. Denote by N_t^f and N_t^x the measure of households employed in agriculture and industry. Thus,

$$N_t = N_t^f + N_t^x. \tag{1}$$

Both types of households are uniformly distributed around the unit circle.

Preferences. A household derives utility from the number of children it raises, n_t, consumption of the agricultural good, c_{at}, and consumption of the differentiated industrial goods, $\{c_{vt}\}_{v \in V_t}$, where V_t denotes the set of differentiated goods produced at time t. Following the literature on the structural transformation and the demographic transition, each household has an agricultural

[8] A richer version would allow for multiple countries and transportation (or trade) costs. Although this would allow us to analyze the effect of a reduction in transportation (or trade) costs, it would come at the cost of increased analytical complexity.

subsistence constraint, represented by $c_{\bar{a}}$ in the utility function. Departing from the literature on the demographic transition, we assume that household utility does not depend on the quality of children.

A household located at point $\tilde{v}$ on the unit circle has the following Cobb-Douglas preferences:

$$U_{\tilde{v}}(c_{at}, n_t, \{c_{vt}\}_{v \in V_t}) = [(c_{at} - c_{\bar{a}})^{1-\alpha}[g(c_{vt}|v \in V_t)]^{\alpha}]^{\mu}(n_t)^{1-\mu}, \tag{2}$$

where

$$g(c_{vt}|v \in V_t) = \max_{v \in V_t}[\frac{c_{vt}}{1 + d_{v\tilde{v}}^{\beta}}]. \tag{3}$$

The subutility $g(c_{vt}|v \in V_t)$ follows Lancaster (1979) by assuming that each household has a variety of the differentiated good that it prefers above all others. This ideal variety corresponds to the household's location on the unit circle, $\tilde{v}$. The further away an industrial variety v lies from a household's ideal variety, the lower the utility it derives from consuming a unit of variety v. In particular, the quantity of variety v that gives the household the same utility as one unit of its ideal variety $\tilde{v}$ is $1 + d_{v\tilde{v}}^{\beta}$, where $d_{v\tilde{v}}$ denotes the shortest arc distance between v and $\tilde{v}$, and $\beta > 0$ is a parameter that determines how fast a household's utility diminishes with the distance from its ideal variety.[9]

Demographics. Households live for one period. Let n_t^i denote the number of children of a household that is employed in sector $i \in \{f, x\}$.[10] The law of motion for the population is then

$$N_{t+1} = n_t^f N_t^f + n_t^x N_t^x. \tag{4}$$

There is a time cost of rearing children, denoted by τ^i, that depends on the sector $i \in \{f, x\}$ in which the household works. We assume that this time cost is higher in the city than in the countryside, i.e., $\tau^x \geq \tau^f$. This assumption is important for generating the second phase of the demographic transition, characterized by a declining population growth rate. It is consistent with the the historical record. Jones and Tertilt (2006), for example, report that the number of children per woman was higher in non-urban areas and on farms throughout the 19th century, and Williamson (1985) reports that the natural rate of increase for the urban population in 19th century England was lower than in the countryside. The reasons for these regional fertility and population

[9] The expression $1 + d_{v\tilde{v}}^{\beta}$ is known as Lancaster's compensation function.

[10] We need not index the number of children by a household's location on the unit circle because the fertility choice is independent of location. This is also the case for the agricultural good.

differences are multiple. For one, infant mortality was higher in the city on account of unhealthy living conditions, a problem that persisted in the United States until the 1920s and the advent of urban sanitation systems. For another, laws restricting child labor in 19th century England applied only to factory work (Doepke and Zilibotti 2005). [11]

Utility Maximization. The differential cost of rearing children in the city and on the farm implies that household income will be different across sectors in equilibrium, even though households are free to move at the beginning of the period. We therefore distinguish between an agricultural household's income per unit of time worked, y_t^f, and an industrial household's income per unit of time worked, y_t^x. The budget constraint of a household working in sector $i \in \{f, x\}$ is then

$$y_t^i(1 - \tau^i n_t^i) \geq c_{at}^i + \sum_{v \in V} p_{vt} c_{vt}^i \tag{5}$$

Maximizing (2) subject to (5) yields the following first order necessary conditions:

$$\begin{aligned} c_{at}^i &= \mu(1-\alpha)(y_t^i - c_{\bar{a}}) + c_{\bar{a}} && (6) \\ \sum_{v \in V_t} p_{vt} c_{vt}^i &= \mu\alpha(y_t^i - c_{\bar{a}}) && (7) \\ \tau^i n_t^i &= (1-\mu)(1 - \frac{c_{\bar{a}}}{y_t^i}) && (8) \end{aligned}$$

We make assumptions on the technology parameters to ensure that $y_t^i(1 - \tau^i n_t^i) > c_{\bar{a}}$ for all $t \geq 0$.

To further characterize the optimal consumption of the differentiated goods, we exploit the linearity property of the subutility function (3) with respect to the set of differentiated goods. This implies that each agent consumes a single industrial variety. In particular, an agent buys the variety, $v' \in V_t$, that minimizes the cost of an equivalent unit of its ideal variety, $p_{vt}(1 + d_{v\tilde{v}}^\beta)$. Namely,

$$v' = \operatorname{argmin}[p_{vt}(1 + d_{v,\tilde{v}}^\beta) | v \in V_t]. \tag{9}$$

Using (7), a household with ideal variety $\tilde{v}$ working in sector i therefore buys the following quantity of variety v':

$$c_{v't}^i = \frac{\mu\alpha(y_t^i - c_{\bar{a}})}{p_{v't}} \tag{10}$$

Its demand for all other varieties $v \in V_t$ is zero.

[11] Another reason is that it was possible to simultaneously watch children and tend vegetables in the countryside, but not in the city, where factory work dominated. Of course, the higher number of children in rural areas may also have been due to them being able to work more easily on the farm than in the factory (Rosenzweig and Evensen, 1977, Doepke, 2004). In our model, however, children do not participate in the labor force.

3.2 Industrial Sector

The industrial sector is monopolistically competitive, and produces a set of differentiated goods, each with a unique address on the unit circle. As in Lancaster (1979), firms can costlessly relocate on the unit circle. The technology for producing industrial goods uses labor as its only input. The existence of a fixed cost, which takes the form of labor, gives rise to increasing returns. Each firm chooses its price and technology, taking aggregate variables and the choices of other firms as given. There is free entry and exit, so that the number of firms, and varieties, will adjust to ensure all firms make zero profits.

Production. Let Q_{vt} be the quantity of variety v produced by a firm; L_{vt} the units of labor it employs; A_{vt} its technology level, or production process; and κ_{vt} its fixed cost in terms of labor. Then the output in period t of the firm producing variety v is

$$Q_{vt} = A_{vt}[L_{vt} - \kappa_{vt}] \tag{11}$$

Both the fixed labor cost, κ_{vt}, and the technology level, A_{vt}, depend on the firm's rate of process innovation, g_{vt}. In particular, the fixed labor cost is given by

$$\kappa_{vt} = \kappa e^{\phi g_{vt}}. \tag{12}$$

Thus, there are two components to the fixed cost: an innovation cost, represented by $e^{\phi g_{vt}}$, that is increasing in the size of process innovation, g_{vt}, on account that $\phi > 0$, and an operating cost, κ, that is incurred even if there is no process innovation. The firm's technology level, A_{vt}, is given by

$$A_{vt} = (1 + g_{vt})A_{xt}, \tag{13}$$

where A_{xt} is the benchmark technology in period t, taken to be the average technology used by industrial firms in period $t-1$:

$$A_{xt} = \sum_{v \in V_{t-1}} \frac{1}{m_{t-1}} A_{v,t-1}, \tag{14}$$

where m_{t-1} is the number of varieties produced in period $t-1$, i.e., $m_{t-1} = card(V_{t-1})$. Therefore, if $g_{vt} = 0$, so there is no process innovation, a firm uses the industrial benchmark technology, A_{xt}, whereas if $g_{vt} > 0$, the firm uses a technology that is $(1 + g_{vt})$ times greater than the benchmark technology.[12]

[12] Thus, we assume complete intertemporal knowledge spillovers. While the existence of this spillover implies a dynamic inefficiency, it is not important to the points we wish to establish. We make the assumption because it is

Profit Maximization. The fixed operating cost implies that each variety, regardless of the technology used, will be produced by a single firm. In maximizing its profits, each firm behaves non-cooperatively, taking the choices of other firms as given. Profit maximization determines the price and quantity to be sold, the number of workers to be hired, and the technology to be operated. As is standard in models of monopolistic competition, firms take all aggregate variables in the economy as given.[13]

Using (11), the profits of the firm producing variety v, Π_{vt}, can be written as

$$\Pi_{vt} = p_{vt}C_{vt} - w_{xt}[\kappa e^{\phi g_{vt}} + \frac{C_{vt}}{A_{xt}(1+g_{vt})}], \tag{15}$$

where w_{xt} is the wage in the industrial sector, and p_{vt} is the price of variety v.

The problem of the firm producing variety v is to choose (p_{vt}, g_{vt}) to maximize (15), subject to the aggregate demand for its product, C_{vt}. As usual, the profit maximizing price is a markup over the marginal unit cost $w_{xt}/[A_{xt}(1+g_{vt})]$, so that

$$p_{vt} = \frac{w_{xt}}{A_{xt}(1+g_{vt})}\frac{\varepsilon_{vt}}{\varepsilon_{vt}-1}, \tag{16}$$

where ε_{vt} is the price elasticity of demand for variety v:

$$\varepsilon_{vt} = -\frac{\partial C_{vt}}{\partial p_{vt}}\frac{p_{vt}}{C_{vt}}.$$

The first order necessary condition associated with the choice of technology, g_{vt}, is

$$-\phi\kappa e^{\phi g_{vt}} + \frac{C_{vt}}{A_{xt}(1+g_{vt})^2} \leq 0, \tag{17}$$

where the strict inequality case in the above expression corresponds to a corner solution, i.e., $g_{vt} = 0$.

3.3 Farm Sector

The farm sector is perfectly competitive. Farms produce a single, non-storable consumption good, that serves as the economy's numéraire. The farm technology is constant returns to scale, and uses labor and land. There is a measure one of farms.

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not possible to solve for an equilibrium with asymmetric firms using Lancaster's construct. Without the assumption of complete intertemporal knowledge spillovers, new varieties would start out at a lower technology, and hence there would not be a symmetric equilibrium.

[13] In principle this requires firms to be of measure zero, a condition that is not satisfied. See Desmet and Parente (2010) for a discussion of how firms could be made of measure zero, without changing any of the results.

Production. Let Q_{at} denote the quantity of agricultural output of the stand-in farm, and L_{at} the corresponding agricultural labor input. Without loss of generality, we normalize the land owned by the stand-in farm to 1. The production function is Cobb-Douglas in land and labor with a labor share of $1 \geq \theta > 0$, namely,

$$Q_{at} = A_{at} L_{at}^{\theta} \tag{18}$$

Agricultural TFP, A_{at}, grows at a rate $g_{at} > 0$ during period t, so that

$$A_{at+1} = A_{at}(1 + g_{at}). \tag{19}$$

During the Malthusian phase agricultural TFP grows at a constant exogenous rate, $\gamma_a > 0$.[14] However, once the industrial sector starts innovating, we allow for agricultural TFP growth to accelerate. In particular, we assume that

$$g_{at} = \max\{\gamma_a, \frac{A_{xt} - A_{xt-1}}{A_{xt-1}}\} \tag{20}$$

This assumption is meant to capture the large secular rise in the growth rate of agricultural TFP since the *Industrial Revolution*, as documented by Federico (2006). Implicitly, it reflects the importance of innovations in the form of farm equipment and synthetic fertilizers originating in the industrial sector.[15]

Profit Maximization. The profit maximization problem of farms is straightforward, as they are price takers. The profit of the stand-in farm is

$$\Pi_{at} = A_{at} {L_{at}}^{\theta} - w_{at} L_{at} \tag{21}$$

where w_{at} is the agricultural wage rate. Farms choose L_{at} to maximize equation (21). This yields the standard first order condition

$$w_{at} = \theta A_{at} (L_{at})^{\theta - 1}. \tag{22}$$

Total profits (or land rents) are thus,

$$\Pi_{at} = (1 - \theta) A_{at} (L_{at})^{\theta} \tag{23}$$

[14] To be consistent with the historical record of a slowly increasing population during the pre-industrial era, agricultural TFP growth must be positive if there are decreasing returns to land.

[15] Alternatively, though at the cost of substantial complexity, the same qualitative results could be obtained by having farms use industrial goods as intermediate inputs, rather than assuming that agricultural TFP growth depends on technological progress in the industrial sector. As technological improvement in industry lowers the relative price of industrial goods, farms would use more industrial intermediate inputs, thereby increasing farm labor productivity. Results for this setup are available from the authors upon request.

and profits per unit of time worked, π_{at}, are

$$\pi_{at} = (1-\theta)A_{at}(L_{at})^{\theta-1}. \tag{24}$$

Profits (or land rents) are rebated to the farm households in proportion to their time worked. Hence, total income of a farm household per unit of time worked is the sum of wages per unit of time worked and profits per unit of time worked:

$$y_t^f = A_{at}(L_{at})^{\theta-1} \tag{25}$$

Urban households, therefore, do not receive any farm profits. Their income per unit of time worked is

$$y_t^x = w_{xt} \tag{26}$$

4 Equilibrium

As is standard in this literature, we focus exclusively on symmetric Nash equilibria. In such an equilibrium, all firms use the same technology, charge the same price, and are equally spaced around the unit circle. This section defines a symmetric Nash equilibrium for our economy, and explores the limiting properties of the equilibrium. It consists of three parts. In the first part, we derive the aggregate demand for each good in the symmetric case, and use this to simplify the first order profit maximization conditions. In the second part, we define a symmetric equilibrium. In the last part, we establish a set of parametric restrictions that ensure that the economy converges to a balanced growth path.

4.1 Aggregate Demand

We first determine the aggregate demand for each industrial good. Demand comes from both types of households. Since in a symmetric Nash equilibrium all varieties produced are equally spaced around the unit circle, aggregate demand for a given variety depends only on the locations and the prices of its closest neighbors to its right and its left on the unit circle. Let d_t denote the distance between two neighboring varieties in period t. This distance is inversely proportional to the number of varieties, m_t, namely,

$$d_t = \frac{1}{m_t}. \tag{27}$$

Since the nearest competitors to the right and to the left of the firm producing variety v are each located at the same distance d_t from it, we do not need to differentiate between them, and thus denote each competitor by v_c and their prices by p_{ct}.

To begin, we derive the aggregate demand of agricultural households for variety v. The first step is to identify the location of the household on the unit circle that is indifferent between buying variety v and variety v_c. Recall that each household will buy that variety for which the unit cost of an equivalent unit of its ideal variety is lowest. Thus, the agricultural household that is indifferent between buying varieties v and v_c is the one whose cost of a quantity equivalent to one unit of its ideal variety in terms of v equals the cost of a quantity equivalent to one unit of its ideal variety in terms of v_c. Consequently, the agricultural household that is indifferent between v and v_c is the one located at distance d_{vt} from v, where

$$p_{ct}[1 + (d - d_{vt})^{\beta}] = p_{vt}[1 + d_{vt}^{\beta}]. \tag{28}$$

Given this indifference condition applies to agricultural households both to the right and to the left of v, the uniform distribution of agricultural households around the unit circle implies that a share $2d_{vt}$ of them consumes variety v. Since each household spends $\mu\alpha(y_t^f - c_{\bar{a}})$ on the industrial good, the total demand for v by agricultural households is

$$C_{vt}^f = 2d_{vt}N_t^f c_{vt}^f = \frac{2d_{vt}N_t^f \mu\alpha(y_t^f - c_{\bar{a}})}{p_{vt}}. \tag{29}$$

By analogy, total demand by industrial households is

$$C_{vt}^x = 2d_{vt}N_t^x c_{vt}^x = \frac{2d_{vt}N_t^x \mu\alpha(y_t^x - c_{\bar{a}})}{p_{vt}}. \tag{30}$$

Given that all firms are spaced evenly in the symmetric equilibrium, it follows that

$$2d_{vt} = d_t. \tag{31}$$

Aggregate demand for variety v, C_{vt}, is the sum of (29) and (30). Hence,

$$C_{vt} = d_t(N_t^f c_{vt}^f + N_t^x c_t^x) = \frac{d_t\mu\alpha(y_t^f N_t^f + y_t^x N_t^x - N_t c_{\bar{a}})}{p_{vt}} \tag{32}$$

With this demand in hand, we can solve for the price elasticity in a symmetric Nash equilibrium. This involves three steps. First, from (32) it is easy to show that

$$-\frac{\partial C_v}{\partial p_v}\frac{p_{\ v}}{C_v} = 1 - \frac{\partial d_v}{\partial p_v}\frac{p_v}{d_v} \tag{33}$$

Next, by taking the total derivative of the indifference equation (28) with respect to p_{vt}, we solve for $\partial d_{vt}/\partial p_{vt}$, and substituting this partial derivative in (33) yields

$$\varepsilon_{vt} = 1 + \frac{[1 + d_v^{\beta}]p_v}{[p_v\beta d_v^{\beta-1} + p_c\beta(d - d_v)^{\beta-1}]d_{\ v}} \tag{34}$$

Finally, we invoke symmetry, i.e., $p_{vt} = p_{ct}$ and $2d_{vt} = d_t$, so that (34) reduces to

$$\varepsilon_{vt} = 1 + \frac{1}{2\beta}(\frac{2}{d_t})^{\beta} + \frac{1}{2\beta} \tag{35}$$

Thus, as the number of varieties increases, the price elasticity of demand increases.

Aggregate demand for the agricultural good is easy to determine. Individual household's demand, (6), implies that aggregate demand is

$$C_{at} = \mu(1-\alpha)(y_t^f N_t^f + y_t^x N_t^x - N_t c_{\bar{a}}) + N_t c_{\bar{a}} \tag{36}$$

4.2 Symmetric Equilibrium

We next define a symmetric Nash Equilibrium for our economy. Because the decisions of households, industrial firms and farms are all static, the dynamic equilibrium for the model economy is essentially a sequence of static equilibria that are linked through the laws of motion for the population, the benchmark technology in the industrial sector, and TFP in the farm sector.

As is standard, the equilibrium must satisfy profit maximization, utility maximization, and market clearing conditions. It must also be the case that each household is indifferent between working in the farm sector and working in the industrial sector. More specifically, for a household with ideal variety v the utility associated with consumption and children should be the same across sectors:

$$U(c_{vt}^f, c_{at}^f, n_t^f) = U(c_{vt}^x, c_{at}^x, n_t^x). \tag{37}$$

Another condition requires that firms in the industrial sector earn zero profits in equilibrium. This is a consequence of there being free entry. Thus,

$$p_{vt}Q_{vt} - w_{xt}[\kappa e^{\phi g_{vt}} + \frac{Q_{vt}}{A_{xt}(1+g_{vt})}] = 0 \tag{38}$$

This condition effectively determines the number of varieties and the distance between varieties.

The zero profit condition (38), together with mark-up equation (16) and the elasticity equation in the symmetric equilibrium (35), provide the key to understanding the positive relation between market size and firm size. From the elasticity expression (35) it is apparent that as the number of varieties increases, and the distance between firms decreases, the price elasticity of demand increases. This result is easy to understand: by increasing the number of varieties, the unit circle becomes more crowded, making neighboring varieties more substitutable. From the price expression (16), it follows that the greater elasticity leads to tougher competition, reducing the markup. The zero profit condition (38) then implies that the size of firms, in terms of production, must increase: given the same fixed cost, a firm must sell a greater quantity of units in order to break even. As we will see later, larger firms find it easier to bear the fixed cost of innovation, leading to a positive relation between market size and technological progress.

We now define the dynamic *Symmetric Equilibrium.*

Definition of Symmetric Equilibrium. *A Symmetric Equilibrium is a sequence of household variables* $\{c^f_{vt}, c^f_{at}, n^f_t, y^f_t, N^f_t, c^x_{vt}, c^x_{at}, n^x_t, y^x_t, N^x_t\}$*, a sequence of farm variables* $\{Q_{at}, L_{at}, \pi_{at}\}$*, a sequence of industrial firm variables* $\{Q_{vt}, L_{vt}, p_{vt}, g_{vt}, \varepsilon_{vt}, A_{vt}\}$*, and a sequence of aggregate variables* $\{V_t, w_{xt}, m_t, w_{at}, d_t, N_t, A_{xt}, A_{at}\}$ *that satisfy*

(i) utility maximization conditions given by (6), (7) and (8).

(ii) farm profit maximization conditions given by equations (18), (22), (24) and (25).

(iii) industrial profit maximization conditions given by (11), (13), (12), (26), (16), (17), and (35)

(iv) market clearing conditions

(a) industrial goods market: equation (11) = equation (32)

(b) industrial labor market:

$$m_t L_{vt} = N^x_t(1 - \tau^x n^x_t) \tag{39}$$

(c) farm goods market: equation (18) = equation (36)

(d) farm labor market:

$$L_{at} = N^f_t(1 - \tau^f n^f_t) \tag{40}$$

(v) aggregate laws of motion for A_{xt} *given by (14); for* N_t *given by (4); and for* A_{at} *given by (19) and (20)*

(vi) zero profit condition of industrial firms given by (38)

(vii) indifference condition of households given by (37)

(viii) population feasibility given by (1)

4.3 Properties

We conclude this section by addressing the limiting properties of the model, namely, whether the economy converges to a balanced growth path. We do this because economic growth in the leading industrialized nations has been fairly constant over the 20th century.

Proposition 1. *The economy converges to a balanced growth path with constant technological progress in industry provided that (i)* $(\frac{1-\mu}{\tau^f}-1)(\frac{1-\alpha}{\alpha})(\frac{\tau^x}{\tau^f})^{\frac{1-\mu}{\mu}}+(\frac{1-\mu}{\tau^f}-1)=0$, *(ii)* $g_{at}>0$ *for all* t, *and (iii)* θ *is sufficiently close to 1.*

Proof. We start by recalling three equilibrium conditions: at all times utility should be equal across farming and industrial households,

$$(y_t^f-c_{\bar{a}})^\mu(\frac{1-c_{\bar{a}}/y_t^f}{\tau^f})^{1-\mu}=(y_t^x-c_{\bar{a}})^\mu(\frac{1-c_{\bar{a}}/y_t^x}{\tau^x})^{1-\mu} \tag{41}$$

and income should equal expenditure in both sectors,

$$y_t^f N_t^f(1-\tau^f n_t^f)=\mu(1-\alpha)(y_t^f N_t^f+y_t^x N_t^x)+(1-\mu(1-\alpha))c_{\bar{a}}(N_t^f+N_t^x) \tag{42}$$

$$y_t^x N_t^x(1-\tau^x n_t^x)=\mu\alpha(y_t^f N_t^f+y_t^x N_t^x)-\mu\alpha c_{\bar{a}}(N_t^x+N_t^x). \tag{43}$$

We now show that y_t^f in the limit goes to infinity. To do so, it suffices to show that the growth in y_t^f is strictly positive in each period. Since $y_t^f=A_{at}L_{at}^{\theta-1}$, this amounts to showing that

$$g_{at}-(1-\theta)\frac{\dot{L}_{at}}{L_{at}}>0. \tag{44}$$

Expression (8) implies that population growth is finite, and therefore growth in the hours worked in agriculture is also finite, so that if θ is close enough to 1, expression (44) is satisfied.

Since $\tau^f>\tau^x$, condition (41) implies that if y_t^f goes to infinity in the limit, then y_t^x also goes to infinity in the limit. This, in turn, implies that in the limit both types of households have a constant (though different) number of kids:

$$n^i=\frac{1-\mu}{\tau^i} \tag{45}$$

where $i\in\{f,x\}$. As the number of children is constant in the limit, the utility indifference condition (41) implies that

$$\frac{y^f}{y^x}=(\frac{\tau^f}{\tau^x})^{\frac{1-\mu}{\mu}}. \tag{46}$$

Moreover, if we divide equation (42) by (43), it follows that in the limit

$$\frac{y^f N^f}{y^x N^x}=\frac{1-\alpha}{\alpha}. \tag{47}$$

Substituting (46) into (47) then gives the following limit expression:

$$\frac{N^f}{N^x} = \frac{1-\alpha}{\alpha}(\frac{\tau^x}{\tau^f})^{\frac{1-\mu}{\mu}}. \tag{48}$$

Thus, the shares of agricultural and industrial households converge to fixed numbers. This together with (45), implies that in the limit population growth is constant. Substituting (48) and (45) into the expression for population growth, $(n^f-1)N^f/N + (n^x-1)N^x/N$ gives the following expression:

$$(\frac{1-\mu}{\tau^f}-1)\frac{\frac{1-\alpha}{\alpha}(\frac{\tau^x}{\tau^f})^{\frac{1-\mu}{\mu}}}{1+\frac{1-\alpha}{\alpha}(\frac{\tau^x}{\tau^f})^{\frac{1-\mu}{\mu}}} + (\frac{1-\mu}{\tau^x}-1)\frac{1}{1+\frac{1-\alpha}{\alpha}(\frac{\tau^x}{\tau^f})^{\frac{1-\mu}{\mu}}} \tag{49}$$

Therefore, if $(\frac{1-\mu}{\tau^f}-1)(\frac{1-\alpha}{\alpha})(\frac{\tau^x}{\tau^f})^{\frac{1-\mu}{\mu}} + (\frac{1-\mu}{\tau^f}-1) = 0$, as stated in condition (ii), population growth converges to zero. With a constant population, a constant number of children in each sector, and a constant share of households employed in the industrial sector in the limit, it follows that the total number of hours worked in industry also converges to a constant.

It is now easy to show that in the limit g_v is a constant. The case where $g_v = 0$ is trivial. Thus, we focus on the case of an interior solution. The zero profit condition, $Q_v = \kappa e^{\phi g_v} A_x(1+g_v)(\varepsilon-1)$, together with the first order condition for technological progress, (17), implies that g_v is a positive function of the price elasticity of demand:

$$g_v = \frac{\varepsilon_v - 1}{\phi} - 1. \tag{50}$$

Since the total production of each firm is $\kappa e^{\phi g_v} A_x(1+g_v)(\varepsilon-1)$ and the total number of hours worked in industry is μN^x in the limit, it follows that the number of firms is $m = \mu N^x/(\kappa e^{\phi g_v}\varepsilon)$, where $m = 1/d$. Substituting into (35) gives

$$\varepsilon = 1 + \frac{1}{2\beta}(\frac{2\mu N^x}{\kappa e^{\phi g_v}\varepsilon})^\beta + \frac{1}{2\beta}. \tag{51}$$

Now re-write (51) as

$$2\beta\varepsilon^{\beta+1} - (2\beta+1)\varepsilon^\beta - (2\mu N^x/\kappa e^{\phi g_v})^\beta = 0 \tag{52}$$

and take the total derivative of this expression with respect to g_v. This yields

$$\frac{\partial\varepsilon}{\partial g_v} = -\frac{\beta(2\mu N^x)^\beta\kappa^{-\beta}\phi e^{-\phi g_v\beta}}{2\beta(\beta+1)\varepsilon_t^\beta - (2\beta+1)\beta\varepsilon^{\beta-1}}. \tag{53}$$

From (35) we know that $\varepsilon > 1$, so that this derivative (53) is strictly negative. Given that (51) implies that ε is decreasing in g_v and (50) implies that ε is increasing in g_v, and given that N^x is constant in the limit, there is a unique, and constant, g_v, and thus a unique, and constant, ε.

Therefore, if N^x is constant, g_v is also constant. The economy therefore converges to a balanced growth path with constant growth in the industrial sector.

□

In Proposition 1 we have shown that if population growth converges to zero, the economy converges to a balanced growth path with constant technological progress in the industrial sector. (This constant rate of technological progress may be zero.) Next, we show that the rate of technological progress in the limit is an increasing function of the balanced growth path population, N, and a decreasing function of the cost of innovation, ϕ.

Proposition 2. *Technological progress in the balanced growth path is an increasing function of population, N, and a decreasing function of the innovation cost parameter, ϕ.*

Proof. As argued in the proof of Proposition 1, on the balanced growth path, expressions (50) and (51) determine the rate of technological progress and the price elasticity of demand. Expression (50) does not depend on N^x, whereas expression (51) does. To see this, re-write (51) as (52) and totally differentiate, keeping g_v fixed. This gives

$$\frac{\partial \varepsilon}{\partial N^x} = \frac{(2\mu/\kappa e^{\phi g_v})^\beta \beta (N^x)^{\beta-1}}{2\beta(\beta+1)\varepsilon^\beta - (2\beta+1)\beta\varepsilon^{\beta-1}} \tag{54}$$

Since $\varepsilon > 1$, the above partial derivative is strictly positive, so that an increase in N^x leads to a greater elasticity of demand for any given g_v. Recall that expression (50) implies that the elasticity is upward sloping in g_v, whereas expression (51) implies that the elasticity is downward sloping in g_v. This, together with the fact that a greater value of N^x causes an upward shift in expression (51), allows us to conclude that g_v is increasing in N^x. Since N^x is a fixed share of N, g_v is therefore also increasing in the size of the population.

To show that g_v is decreasing in ϕ, we use a similar argument. Re-write (51) as (52) and totally differentiate with respect to ϕ, keeping N^x and g_v constant. This gives

$$\frac{\partial \varepsilon}{\partial g_v} = -\frac{\beta(2\mu N^x)^\beta \kappa^{-\beta} g_v e^{-\phi g_v \beta}}{2\beta(\beta+1)\varepsilon^\beta - (2\beta+1)\beta\varepsilon^{\beta-1}} \tag{55}$$

Since $\varepsilon > 1$, the above partial derivative is strictly negative, so that an increase in ϕ leads to a smaller elasticity of demand for any given g_v. By analogy with the above argument, this implies that g_v is decreasing in ϕ. □

The intuition for the positive relation between the size of the limit population and the balanced growth path rate of technological progress is straightforward. A greater population leads to a larger number of households employed in the industrial sector. The greater size of the industrial sector, and the larger number of varieties produced, implies lower mark-ups, and tougher competition. To break even, industrial firms must become larger. These larger firms then endogenously choose to innovate more. This is obvious from the first order condition on technology choice, (17), which exhibits two effects: an increase in innovation raises a firm's fixed cost and lowers its marginal cost. The first (negative) effect is independent of firm size, whereas the second (positive) effect is increasing in firm size. As a result, larger firms innovate more.

Propositions 1 and 2 lead to a number of conclusions. Starting off in a situation with no technological progress in industry, two situations can arise. If population reaches the critical size for take-off before population growth converges to zero, we will get an industrial revolution, and the economy will converge to a balanced growth path with strictly positive technological progress in industry. However, if population growth converges to zero before that critical size is reached, we have an industrialization trap, and the industrial sector never innovates. As Proposition 2 suggests, this industrialization trap becomes increasingly likely, the higher is ϕ. This is easy to see when considering the extreme case of ϕ being infinite. Then obviously there will never be any take-off.[16]

5 Numerical Experiments

In this section, we calibrate the model to the historical record of England over the period 1300-2000, and use the calibrated structure to examine how the timing of the industrial revolution is affected by a number of factors emphasized by other researchers as being important for understanding why England was the first country to develop.

5.1 Calibration

The calibration strategy is to assign parameter values so that the model equilibrium is characterized initially by a Malthusian-like era and in the limit by a modern growth era. Empirically, the key observations of the Malthusian era targeted in the calibration are: (i) a constant living standard, (ii) a constant rate of population growth, and (iii) a dominant share of agricultural activity in the economy. Empirically, the key observations of the modern growth era targeted in the calibration are: (i) a constant, positive rate of growth of per capita GDP, and (ii) a dominant share of industrial activity in the economy. Theoretically, for the model to generate a modern growth era, the

[16]We do not call this a development trap because unless we make assumptions that γ_a in equation (21) goes to zero in the limit, then there will be increases in agricultural output per person in the industrialization trap.

population growth rate must converge to zero in the limit. This is another key restriction in the calibration exercise.

Before assigning parameters, it is necessary to identify the empirical counterpart of a model period, and starting date. Given that households live for one period during which they raise their offspring, it is reasonable to interpret a period as the time that elapses between generations. In models that employ a two-period generational construct, a period is typically assigned a length of 35 years. However, since our model is one of the last millennium and as life expectancies were far shorter before the 20th century, we choose 25 years for the period length. The first model period is identified with the year 1300. This choice is primarily motivated by data availability.

Table 1: Parameter values

1. Population	
$N_0 = 346$	start of industrial revolution in 1750
2. Industrial technology parameters	
$A_{x0} = 1$	normalization
$\kappa = 3.7$	median percentage of ratio of non-production to production workers in US manufacturing outside central offices (Berman et al., 1994)
$\phi = 4.5$	limiting growth of per capita GDP between 1-5-2.0% (Maddison, 2001)
3. Agricultural technology parameters	
$A_{a0} = 11.11$	constant agricultural living standard in pre-1500 era
$\gamma_a = 0.0095$	pre-1500 average annual population growth rate of .025% (Maddison, 2001)
$\theta = 0.71$	1700 labor share in agriculture estimate (Clark, 2000b, Hayami and Ruttan, 1971)
4. Preference parameters	
$c_{\bar{a}} = 1.55$	agricultural share of employment in 1600 (Allen, 2000)
$\alpha = 0.98$	2% limiting share of agriculture's share of employment (Mitchell, 1988)
$\mu = 0.9125$	total fertility rate in 2007 for London of 1.80.
$\beta = 0.50$	mark-up estimates between 5-15% in the limit (Jaimovich and Floetotto, 2008)
5. Child rearing parameters	
$\tau^f = 0.021$	zero population growth growth in the limit
$\tau^x = 0.095$	estimates between 7.5-15% per household (de la Croix and Doepke, 2004)

Table 1 lists the parameter values and provides brief comments on how each was assigned. A few additional words are warranted in the case of some of the parameters. First, the start of the *Industrial Revolution* in the calibration does not correspond to the first period in which industrial firms innovate. Instead, it corresponds to the model period(s) that is followed by a rapid acceleration in the growth rate of per capita GDP and a rapid movement of employment into industry. This is relevant for how the initial population value is set. Second, the value of the rural child rearing time cost, τ^f, is not based on independent estimates of this cost from time-use studies, but rather is set so population growth is zero in the limit. This restriction implies a time rearing cost in the countryside that is roughly 25 percent the city level. Although relatively small, it is in line with estimates by Ho (1979) for rural Philippine households. Finally, the empirical counterpart of the share of agricultural output that goes to labor, namely, the ratio of total farm wages to the sum of total farm rents plus total farm wages, has increased steadily over the last four centuries. Labor's share in agriculture was 67 percent in 1600 according to Clark (200b) and 86 percent in 1950 according to Hayami and Ruttan (1971). As the model does not allow for this secular rise, we set the labor share parameter, θ, to the 1700 trend-value based on a linear interpolation of the 1600 and 1950 estimates.[17]

Figures 1-5 present the equilibrium path for the model economy from 1300 to 2000 along five dimensions: technological progress in the industrial sector, the growth rate of GDP per capita, the growth rate of population, agriculture's share of employment, and the relative price of industrial goods. Where appropriate and available, we plot the real world counterparts for the English economy. Growth rates for both the model economy and England are expressed in annual terms. Data on the growth rates of GDP per capita and population are taken from Maddison (2001). Data on the agricultural share of employment are taken from Allen (2000) for the 1300-1800 period, and thereafter from Mitchell (1988).[18] Data on relative prices are taken from Yang and Zhu (2008) for the 1700-1909 period and extended through 1938 using the Sauerbeck price series in Mitchell (1988).

In terms of population growth, output growth, and agriculture's share of economic activity, the calibrated model matches the historical experience of the English economy extremely well. We emphasize that we did not calibrate the model economy to England's *Industrial Revolution*: we calibrated to the pre-1700 Malthusian era, the post-1950 modern growth era, and a starting date

[17] We have redone the calibration to match the post-1950 estimate, and the equilibrium properties are quantitatively the same.

[18] Clark (2002a) also provide estimates for agriculture's share of employment for the 1500-1700 period for England. They are lower than those of Allen (2000), and can be interpreted as a lower bound. Calibrating to Clark's figures is not a problem. Another alternative is to use the estimates of Allen (2000) for the rural population, rather than the agricultural population. Those figures provide an upper bound.

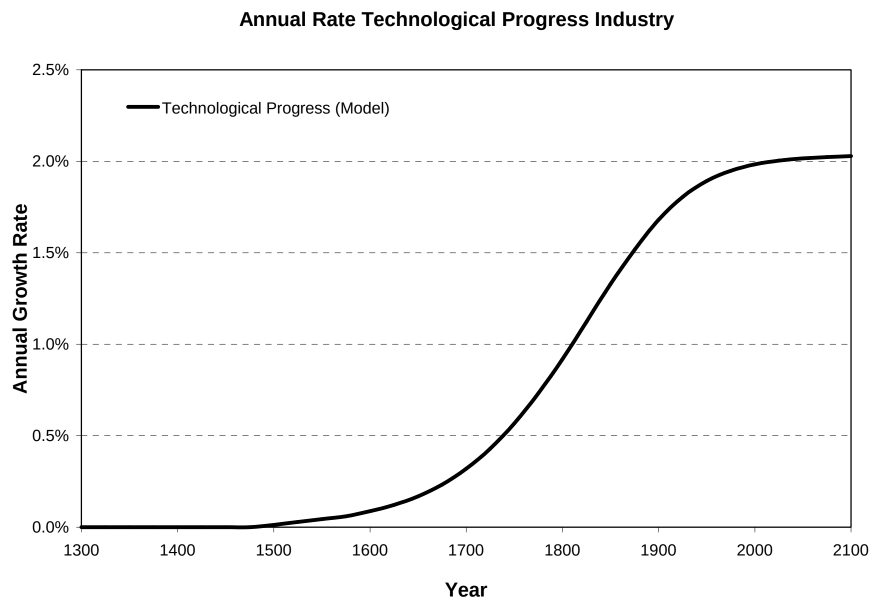

Figure 1: Technological Progress (Benchmark)

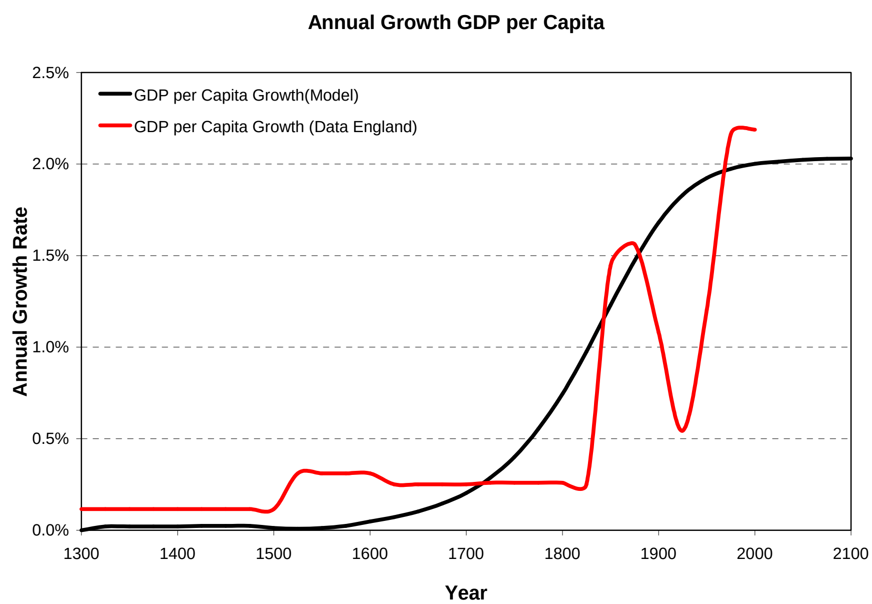

Figure 2: Growth GDP per Capita (Benchmark)

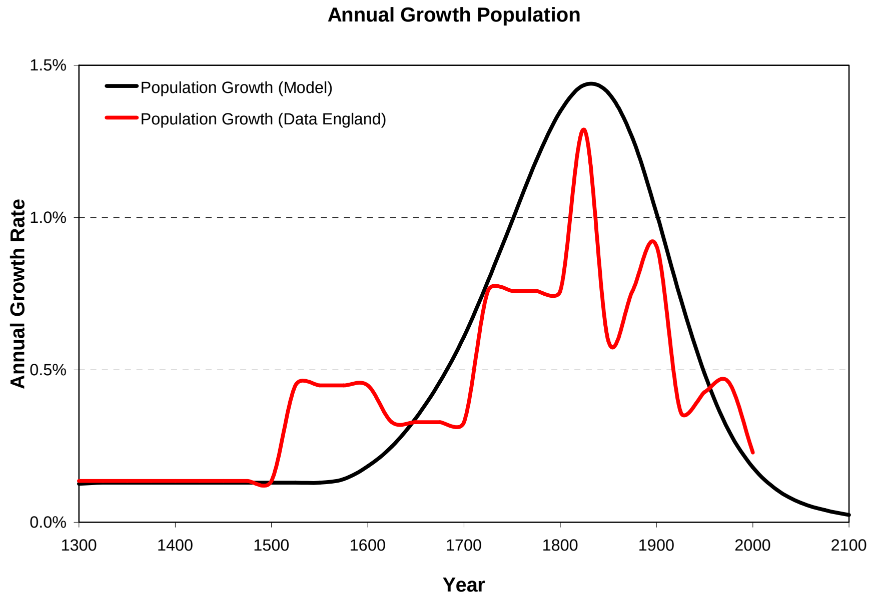

Figure 3: Growth Population (Benchmark)

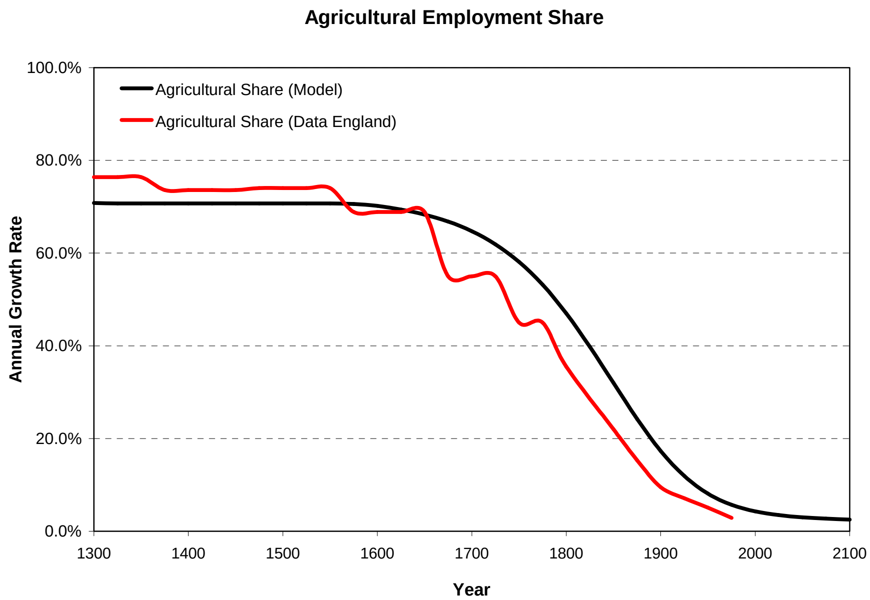

Figure 4: Agricultural Employment Share (Benchmark)

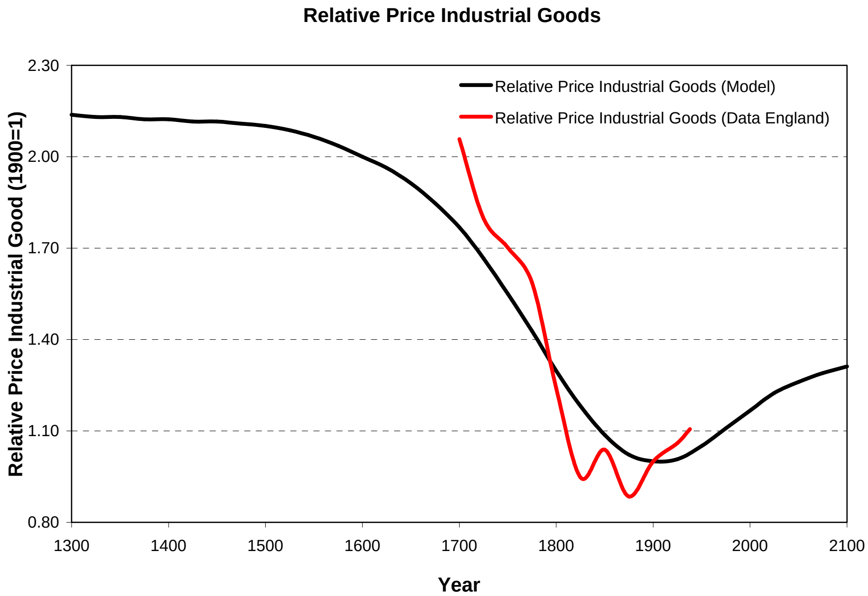

Figure 5: Relative Price Industrial Goods (Benchmark)

of 1750 for the model economy's takeoff. Thus, the model's ability to match the path of England's growth rate of per capita GDP, its population growth rate, and its agriculture's share of employment for the period 1750-1950 so closely represent three successful tests of our theory.

The model also does well in its ability to match the behavior of the relative price of industrial goods over the comparable period for which data are available, 1700 to 1938.[19] As in the data, the relative price of industrial goods is essentially cut in half over this period. As can be seen, the predicted path is non-monotonic. This non-monotonicity reflects the behavior of the ratio of the industrial wage rate to technology, $w_x/[A_x(1+g_v)]$, which affects the price charged by an industrial firm as shown in equation (16). This ratio declines throughout much of the transition period from Malthusian stagnation to modern growth, and then increases slightly before converging to a constant. This pattern arises because the absolute size of the agricultural population initially increases, then decreases, and eventually stabilizes as the economy converges to a constant population. Because land is a fixed factor, this implies that agricultural household income initially grows slower, then faster, and eventually at the same rate than technical progress in industry. Since households must be indifferent between working in both sectors, the evolution of industrial household income

[19] We were not able to extend the British data on relative prices beyond 1938. For the United States, however, the relative price of manufactured goods has shown no secular trend in the 20th century. This is consistent with the model's prediction of a constant relative price in the balanced growth path.

is similar: it first grows more, than less, and eventually at the same rate as technological progress. This explains the non-monotonic behavior of this ratio, and the relative price of industrial goods.

In terms of other relevant statistics, the calibrated model predicts nearly a 400 percent increase in firm size, a 300 percent increase in the number of varieties, and a 50 percent decline in the mark-up over the 1300-2000 period. Where comparable data are available, the model gets the trends right, but tends to underpredict their magnitudes. For example, in the U.S. economy, the average size of manufacturing establishments increased by a factor of 5 between 1870 and 1940, whereas the model predicts a two-fold increase in establishment size over the same period. Likewise, for the English economy, the mark-up declined by 67 percent between 1870 and 1985 according to Ellis (2006), whereas the model predicts a 7 percent decline.

In summary, the model does very well at predicting the main features of England's *Industrial Revolution*, in particular, its demographic transition, its structural transformation, and its rate of growth. It is less successful in its ability to quantitatively match secondary features, such as trends in firm size, product variety, and mark-ups. Given the rather stark structure of the model and given the very long and diverse period of analysis, this is not entirely surprising. In light of the overall success of the model, it is informative to investigate how factors that are likely to differ across societies affect the timing of the industrial revolution. This is the subject we analyze next.

5.2 The Timing of the Industrial Revolution

In this section we explore how certain parameters affect the timing of the industrial revolution. Since numerous researchers have emphasized the role of agriculture for long run development, we consider how the start of the industrial revolution is affected by the economy's initial level of agricultural TFP, A_{a0}, and its growth rate, γ_a. Additionally we examine how the economy's take-off is affected by the fixed cost parameters, κ and ϕ. This we do because operating costs and R&D costs can be affected by institutions, and because numerous researchers have argued that institutional developments were critical for England's economic success. Finally, we examine how the economy's take-off is affected by doubling its initial population, N_0, and land endowment. This experiment aims to capture the effect of trade liberalization, another factor that has been strongly emphasized in the literature. While we cannot analyze the effect of trade liberalization in the sense of a small reduction in transportation costs, doubling an economy's initial population and land endowment is equivalent to taking two identical closed economies and completely opening them up to trade.[20]

5.2.1 Agricultural Productivity

How much later would the *Industrial Revolution* in England have occurred if agricultural productivity had been lower? Several researchers, such as T.S. Schultz (1968) and Jared Diamond (1997), have argued that high agricultural productivity is a necessary condition for long-run development. Towards the goal of answering this question, we conduct two experiments. The first of these lowers the value of initial agricultural TFP, A_{a0}, by 8 percent, whereas the second reduces the growth rate of agricultural TFP during the pre-industrial period, γ_a, by 26 percent. The results for process innovation are displayed in Figure 6.[21]

Not surprisingly, a lower starting level of agricultural TFP delays the onset of the industrial revolution. Lower agricultural TFP delays the start of industrialization because population size is smaller in the Malthusian era implying fewer differentiated goods and smaller industrial firms at any date. The size of the delay, 225 years, associated with the 8 percent decline in agricultural TFP, may seem surprising, but it is not. With the calibrated growth rate of agricultural TFP of 0.038 percent per annum in the benchmark case, it takes slightly more than 200 years for agricultural TFP to rise by 8 percent. In other words, the 225 year delay found in this experiment reflects the time it takes agricultural TFP to reach the initial benchmark level. The size of the delay, however, does suggest that a modest increase in agricultural TFP can be extremely important for an economy's takeoff.

For similar reasons, a lower rate of agricultural TFP growth also delays the onset of the industrial revolution. For an annual growth rate of 0.028 percent (instead of 0.038 percent), the start is only delayed by about 75 years. This shorter delay makes sense: to achieve the same accumulated growth as with the benchmark TFP growth of 0.038 percent over 200 years takes about 75 years more with a TFP growth of 0.028 percent.

[20] We do not consider how the starting date responds to a change in the initial population size alone because such a change only causes a temporary departure from the Malthusian steady state. Lowering the population by, say, 10 percent has no effect on the start of the industrial revolution because the system returns to its Malthusian steady state in a few periods.

[21] In these experiments, we adjust the initial population so that the economy continues to display a Malthusian era steady state.

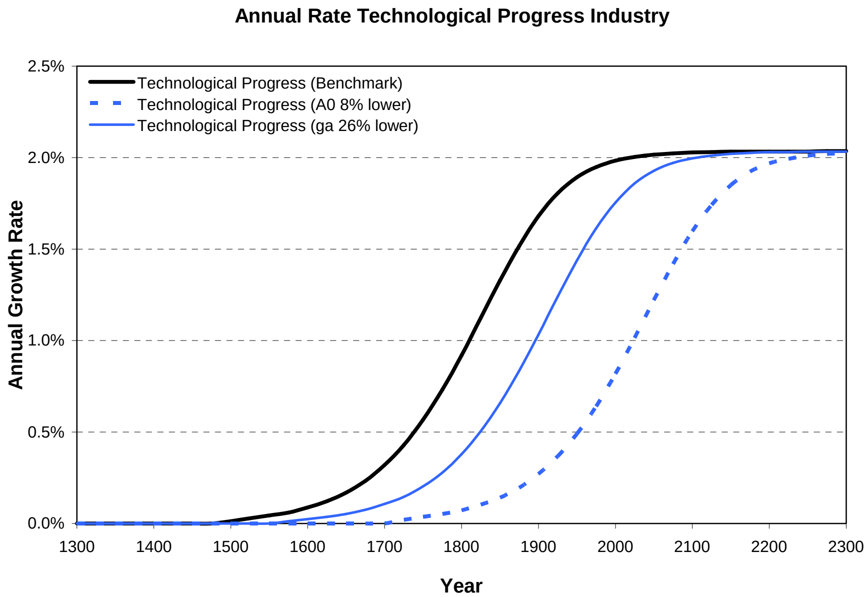

Figure 6: Effect of Lower Agricultural TFP on Timing of Industrial Revolution

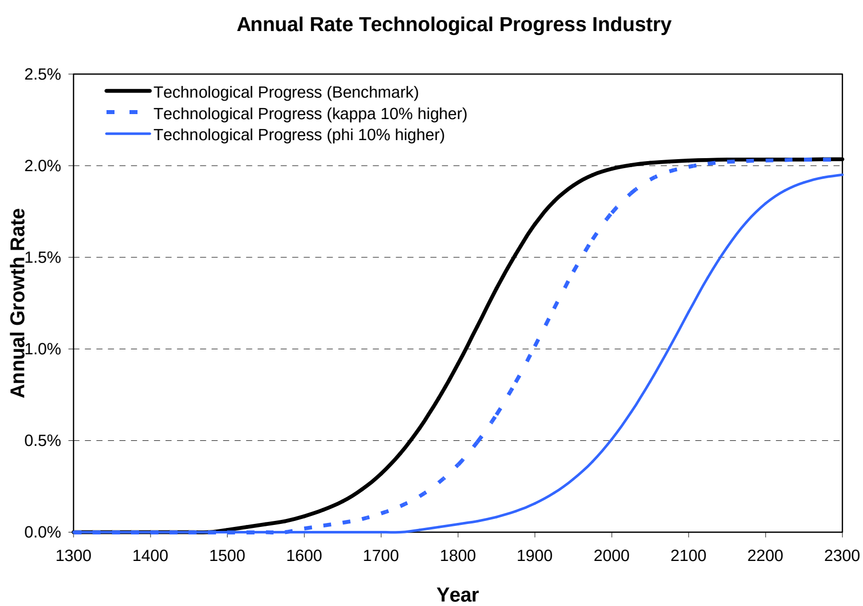

Figure 7: Effect of Worse Institutions on Timing of Industrial Revolution

5.2.2 Policy and Institutions

We next explore how the timing of England's *Industrial Revolution* was affected by institutional factors, a main theme in the research of North and Thomas (1973), North and Weingast (1989), and Ekelund and Tollison (1981). In the real world, the fixed costs firms incur to operate and to innovate depend to a large extent on institutions and policy. We therefore interpret larger values for the fixed cost parameters, κ and ϕ, as worse institutions and policies. Recall that κ is the fixed cost of operating the benchmark technology, whereas ϕ determines how much the fixed cost increases when better technologies are adopted.

Figure 8 shows what happens when we increase each fixed cost parameter separately by 10 percent. In the case of a higher fixed operating cost κ, the industrial revolution is delayed by 100 years; in the case of a 10 percent higher fixed adoption cost ϕ, the industrial revolution is delayed by 250 years. While raising either parameter delays the start of the industrial revolution, the intuition for the delays is different. In the case of κ, worse policies or institutions imply less varieties produced in the economy, meaning the elasticity of demand is lower and innovation is unprofitable. In the case of a 10 percent increase in ϕ, the number of varieties and firm size are unaffected. However, because the cost of process innovation is higher, firms have to be larger to find innovation profitable.

Clark (2003) has criticized institutional based theories on account that changes in British institutions do not time very well with the start of the *Industrial Revolution*. The *Glorious Revolution*, of course, occurred in 1688, but the *Industrial Revolution* did not start for another 100 years. This experiment suggests that Clark's (2003) timing-based argument is not justified. More to the point, our experiment shows that changes in a country's institutions that affect operating and innovation costs are important for the timing of an economy's take-off, even though the date of the take-off may lag these changes by several centuries.

5.2.3 Trade

Both international and intranational trade have been identified by numerous authors, such as Findlay and O'Rourke (2006) and Szostak (1991), as being important for England's early development. In light of this, we end this section by considering the effect of trade on the timing of take-off. While we modeled the economy as closed, we can analyze the case of going from autarky to free trade because in our model a doubling of the economy's population and land mass is equivalent to taking two identical closed economies and opening them up to free trade. Although partial trade

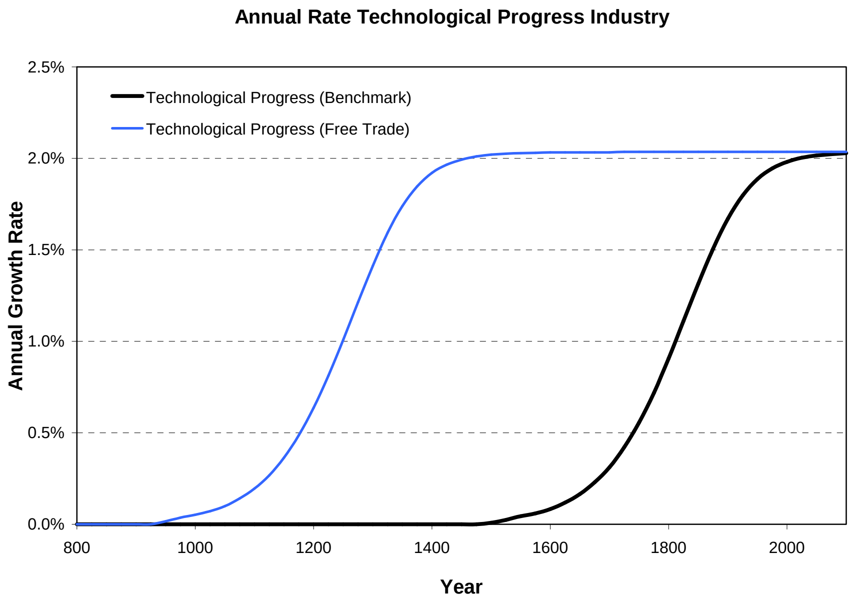

Figure 8: Effect of Trade Liberalization on Timing of Industrial Revolution

liberalization in the sense of an incremental reduction in transport costs may be empirically more relevant, the case of autarky to free trade is, nevertheless, informative and provides an upper bound of the effect of trade on development.

Figure 9 shows the effect of doubling the country's size on the technology choice of industrialized firms. Not surprising, the start of the industrial revolution is dramatically hastened. Whereas 1525 is the the first date a firm lowers its marginal cost in the benchmark, 950 is the first year of process innovation in the open economy case. This experiment suggests an important role played by trade in understanding England's long run development.

6 Conclusion

This paper has put forth a novel theory of the *Industrial Revolution* that is consistent with the pattern of long run development documented by economic historians over the last decades. We have shown that our theory is plausible by calibrating the model to England's long-run development, and by providing empirical support primarily at the firm and industrial level for the mechanism that underlies our theory. We have also examined in the calibrated model the role of various factors emphasized by other researchers as being important for why England was the first country to industrialize. Indeed, a virtue of our theory is that these disparate set of factors all affect the date at which the economy's industrializes by changing the price elasticity of demand.

Clearly, the novelty of the paper lies in the mechanism by which larger markets bring about the *Industrial Revolution*, rather than in the idea that an expansion of markets is critical. Other economists, such as Adam Smith, have argued that market size was important for understanding why England was the first country to experience an industrial revolution. However, Smith and other economists who have made this point have different mechanisms in mind that typically involve increasing returns and specialization. Our mechanism relies on neither.

We see a number of areas where future research will be valuable. On the theoretical side, we could extend the model economy to allow for savings and capital accumulation. In this way, the model can be better matched to the data. On the empirical side, we could compute measures of market potential, that take into account transportation and trade costs, to get better measures of the effective market size. In this way, we could test the theory's main prediction, that market size and not national population per se, is the key variable in determining the date an economy industrializes. Given the success of our theory, these future areas of research are warranted.

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